BOOK OF ABSTRACTS

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1

Evolution equations with superlinear growth

IRENE BENEDETTI
Università di Perugia

ABSTRACT: In this talk we present existence results for mild solutions of the following partial differential equation of parabolic type

$$u_t = \Delta u + h(t, x, u(t, x)) \quad \text{for} \quad (t, x) \in (0, T) \times \Omega$$

coupled with Dirichlet boundary conditions on $\partial\Omega$ and a nonlocal initial condition $u(0, \cdot) = g(u)$ described by a map $g : C([0, T]; L^p(\Omega)) \to L^p(\Omega)$ with $2 \leq p < \infty$, where $\Omega \subset \mathbb{R}^k$ is a bounded domain with C^2-boundary. Here $h\colon [0, T] \times \Omega \times \mathbb{R} \to \mathbb{R}$ is a given map with superlinear growth. The nonlocal condition includes as particular cases the Cauchy multipoint problem, the weighted mean value problem, and the periodic problem. Existence results are obtained by means of a Leray-Schauder continuation principle, transforming the above problem to an ordinary differential equation in the abstract setting given by the Banach space $L^p(\Omega)$. Handling superlinear growth in this context is particularly challenging since the Nemytskii operator associated to the Carathéodory function $h : \Omega \times \mathbb{R} \to \mathbb{R}$ maps the space $L^p(\Omega)$ continuously on itself if and only if h is sublinear, as stated in Vainberg's theorem. We overcome this difficulty exploiting the compactness and the regularity properties of the semigroup generated by the Laplacian operator and constructing a suitable approximation technique.

2

A two-point boundary value problem associated with Hamiltonian systems on a cylinder

ALESSANDRO FONDA
Università di Trieste

ABSTRACT: We prove the existence of multiple solutions for a two-point boundary value problem associated with Hamiltonian systems on a cylinder. Unlike the periodic problem, where the Poincaré-Birkhoff Theorem plays a central role, no twist condition is needed here. This is joint paper with Rafael Ortega.

3

A bifurcation phenomenon for the critical Laplace and p-Laplace equation in the ball

MATTEO FRANCA
Università di Bologna

Abstract: In this talk we show that the number of radial positive solutions of the following critical problem

$$\begin{cases} \Delta_p u(x) + \lambda K(|x|)\, u(x)\, |u(x)|^{q-2} = 0\,, & \\ u(x) > 0 & |x| < 1 \\ u(x) = 0 & |x| = 1. \end{cases}$$

undergoes a bifurcation phenomenon; here $q = \frac{np}{n-p}$, $p > 1$ and $x \in \mathbb{R}^n$. Namely the problem admits one solution for any $\lambda > 0$ if K is steep enough at 0, while it admits no solutions for λ small and two solutions for λ large if K is too flat at 0.

The existence of the second solution is new even in the classical Laplace case. The proofs use Fowler transformation and dynamical systems tools.

4

Birkhoff-Kellogg type result in cones with applications

GENNARO INFANTE
Univerisità della Calabria

ABSTRACT: We present some classical and modern results of Birkhoff-Kellogg type and their application to the solvability of parameter-dependent problems in differential equations.

5

Wavefronts and nonlocal solutions in evolution dynamics

LUISA MALAGUTI
Università di Modena e Reggio Emilia

ABSTRACT: The first part of this talk deals with a scalar reaction-diffusion equation modelling the movement of a biological population with isolated and grouped organisms. Preferences in movements are at the ground of this process. The diffusivity changes sign twice, and we focus our discussion on a bistable reaction term. By means of suitable upper and lower solution, we prove the existence of wavefronts and according to the sign of their speeds we derive some predictions on the survival or extinction of that population.

The second part is about an evolution equation in abstract spaces with nonlocal multivalued associated conditions. The linear part generates a densely defined, not necessarily compact, C0 semigroup of contractions. The nonlocal condition undergoes a restriction which involves the Hausdorff measure of noncompactness. By the homotopic invariance of a topological degree, we provide its solvability and hint its application to some parabolic diffusion equations.

The results come from the papers in preparation

- D. Berti-A. Corli- L.Malaguti. *The role of convection in the existence of wavefronts for biased movements.*
- L. Malaguti-S. Perrotta. *Evolution equations with associated nonlocal multivalued Cauchy problems*

6

Travelling fronts in telegraph-type equations

CRISTINA MARCELLI
Università Politecnica delle Marche

Abstract: We present some results about the existence of travelling wave solutions for the following equation

$$\epsilon^2 u_{tt} + g(u)u_t = (k(u)u_x)_x + \rho(u), \quad 0 \le u \le 1$$

where ρ is a Fisher-type reaction term, k, g are non-negative continuous functions. We also study some properties of the fronts and estimates for the admissible wave speeds.

This is a joint research with F. Papalini (Università Politecnica delle Marche) and M. Cantarini (Università di Perugia).

7

On unbounded solutions for differential equations with mean curvature operator

SERENA MATUCCI
Università di Firenze

ABSTRACT: In this talk, based on a joint work with Zuzana Došlá and Mauro Marini, we will study the existence of nonoscillatory unbounded solution for the nonlinear equation with the mean curvature operator

$$\left(a(t)\frac{x'}{\sqrt{1+x'^2}}\right)' + b(t)F(x) = 0, \quad t \geq t_0 \geq 0, \tag{1}$$

where the functions a,b are positive on $[t_0, \infty)$,

$$\liminf_{t\to\infty} a(t) > 0, \quad \int_{t_0}^{\infty} \frac{dt}{a(t)} = \infty,$$

and the function F is continuous on $\mathbb{R}$, with $F(u)u > 0$ for $u \neq 0$, and satisfies $mu \leq F(u) \leq Mu$ for t large, being $0 < m < M$ suitable constants.

By means of a fixed point approach based on the Schauder linearization device, comparison results with some auxiliary Sturm-Liouville linear equations, and using integral inequalities, we show a necessary and sufficient condition for the existence of solutions of (1) satisfying

$$\lim_{t\to\infty} x(t) = \infty, \quad \lim_{t\to\infty} a(t)x'(t) = 0.$$

8

Periodic solutions to singular problems in special relativity

DUCCIO PAPINI
Università di Udine

ABSTRACT: I'll talk about some results obtained in a joint work with Alberto Boscaggin and Walter Dambrosio (Università di Torino) and take into consideration an equation of the following form:

$$\frac{d}{dt}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\nabla_x V(t,x)$$

where $x = x(t)$ is the position of a particle of mass m at time t, c is the speed of light and V is a T-periodic potential which vanishes at infinity and has some singularities. The paradigm is the gravitational potential generated by a single point mass at rest in the origin, so that:

$$V(t,x) = -\frac{\alpha}{|x|} + U(t,x) \qquad \text{for } |x| \neq 0$$

where $\alpha > 0$ and U is bounded, smooth and T-periodic. With this choices, the equation is a perturbation of Kepler's problem in special relativity. Collisionless T-periodic solutions are critical points of the action functional, that contains the term:

$$\psi(x) = \begin{cases} mc^2 \displaystyle\int_0^T \left(1 - \sqrt{1 - \frac{|\dot{x}|^2}{c^2}}\right) dt & \text{if } \|\dot{x}\|_\infty \leq c, \\ +\infty & \text{otherwise,} \end{cases}$$

which is convex and lower semi-continuous, but not globally differentiable. I'll show the details of the problem of existence and multiplicity of T-periodic solutions in dimension 2, where we make use of an extension of the min-max principle developed by Livrea and Marano (*Adv. Differential Equations* **9** (2004), 961–978) to the case of singular functionals, and of the topology induced by the singularities of the potential on the space of collisionless, T-periodic and Lipschitz orbits. If time allows, I'll give some hints on the problem in dimension 3.

9

Existence and global bifurcation of periodic solutions to second order retarded functional differential equations

M. PATRIZIA PERA
Università di Firenze

Abstract: In this talk, I will present some results on the existence and global bifurcation of T-periodic solutions to second order retarded functional differential equations defined on boundaryless smooth manifolds. Both cases of a topologically nontrivial compact manifold (e.g., an even dimensional sphere) and of a possibly noncompact constraint will be considered. A Rabinowitz-type global bifurcation result as well as a Mawhin-type continuation principle will be deduced. In the scalar case, I will describe some multiplicity results recently obtained with Alessandro Calamai and Marco Spadini where a delay-type functional dependence involving a gamma probability distribution is considered.

The approach used is topological and is based on the fixed point index theory and on degree methods.

10

Asymptotic stability of solutions of differential equations with distributed delay

PAOLA RUBBIONI
Università di Perugia

ABSTRACT: We discuss the asymptotic stability of solutions of the parametric differential equation arising from population dynamics models

$$\frac{\partial u}{\partial t}(t,x) = -b(t,x)u(t,x) + g\left(t, u(t,x), \delta_T(t,x,u)\right),\ t \geq t_0,\ x \in [0,1] \quad (2)$$

where

$$\delta_T(t,x,u) = \int_{-T}^{0} u(t+\theta, x)d\theta \quad \text{or} \quad \delta_T(t,x,u) = \int_{t_0}^{t} \frac{e^{-(t-s)/T}}{T} u(s,x)\,ds$$

is a distributed delay ($T > 0$). In the first case, it means that at every time t the system has memory of the evolution of the state up to that moment t itself for a past of fixed amplitude T. In the second, a spanning effect is provided by means of a memory kernel given by the exponential distribution of probability $\mathcal{k}(\tau) = \frac{e^{-\tau/T}}{T}$. It assigns a greater weight to the most recent events, increasingly fading the influence of those further away in time. In both cases, the positive number T provides the width of the action of the delay: the larger T, the more the system's memory is extended to past events affecting its present state. Inasmuch as the process is set on the whole half-line, the number T can be chosen arbitrarily large.

Depending on the kind of delay, we see equation (2) as a particular case of the semilinear differential equation with functional delay in Banach spaces

$$y'(t) = A(t)y(t) + f\left(t, y(t), y_t\right),\ t \geq t_0, \quad (3)$$

where y_t stands for the function $y_t(\theta) = y(t+\theta)$, $\theta \in [-\tau, 0]$, or of the semilinear integro-differential equation

$$y'(t) = A(t)y(t) + f\left(t, y(t), \int_{t_0}^{t} k(t,s)y(s)ds\right),\ t \geq t_0, \quad (4)$$

where k is a continuous real function.

The results are achieved on (3) and (4) by combining iterative methods and fixed point theorems for condensing maps, and then made to fall back on (2).