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Corrigendum

Corrigendum to "Infinitely many solutions to singular convective Neumann systems with arbitrarily growing reactions" [J. Differ. Equ. 271 (2021) 849–863]

U. Guarnotta, S.A. Marano *

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Abstract

We give a correct formulation of Theorems 4.2-4.3 in [1].

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Notation is the same as that adopted in [1]. Due to a technical mistake, the proofs of [1, Theorems 4.2-4.3] are incorrect. However, their conclusions still hold true provided a further condition (see (S_1)–(S_2) below) on the sign of nonlinearities is assumed. For the reader's convenience, here, we give the amended version of the whole Section 4.

4. Infinitely many solutions

4.1. The sub-linear case

We make the hypotheses below.

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* Corresponding author.
E-mail address: marano@dmi.unict.it (S.A. Marano).

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(F_1') There exist $\alpha_1 < 0 < \beta_1$, $\gamma_1, \delta_1 \in [0, p-1)$, and $a_1, b_1, c_1 \in L^\infty(\Omega)$ such that

$$|f(x,s,t,\xi_1,\xi_2)| \leq a_1(x) s^{\alpha_1} t^{\beta_1} + b_1(x)(|\xi_1|^{\gamma_1} + |\xi_2|^{\delta_1}) + c_1(x)$$

for all $(x,s,t,\xi_1,\xi_2) \in \Omega \times (0,+\infty)^2 \times \mathbb{R}^{2N}$.

(G_1') There exist $\beta_2 < 0 < \alpha_2$, $\gamma_2, \delta_2 \in [0, q-1)$, and $a_2, b_2, c_2 \in L^\infty(\Omega)$ such that

$$|g(x,s,t,\xi_1,\xi_2)| \leq a_2(x) s^{\alpha_2} t^{\beta_2} + b_2(x)(|\xi_1|^{\gamma_2} + |\xi_2|^{\delta_2}) + c_2(x)$$

for all $(x,s,t,\xi_1,\xi_2) \in \Omega \times (0,+\infty)^2 \times \mathbb{R}^{2N}$.

(S_1) There exist $\{h_n\}_n, \{\hat{h}_n\}_n, \{k_n\}_n, \{\hat{k}_n\}_n, \{C_n\}_n \subseteq (0,+\infty)$, with $C_n \to +\infty$, satisfying $h_n < k_n < h_{n+1}$, $\hat{h}_n < \hat{k}_n < \hat{h}_{n+1}$, and

$$\begin{aligned} &f(x,k_n,t,\xi_1,\xi_2) \leq 0 \leq f(x,h_n,t,\xi_1,\xi_2), \\ &g(x,s,\hat{k}_n,\xi_1,\xi_2) \leq 0 \leq g(x,s,\hat{h}_n,\xi_1,\xi_2) \end{aligned} \tag{S'}$$

for all $(x,s,t,\xi_1,\xi_2) \in \Omega \times [h_n,k_n] \times [\hat{h}_n,\hat{k}_n] \times B_{\mathbb{R}^N}(C_n)^2$, $n \in \mathbb{N}$. Further,

$$\begin{aligned} &\|a_1\|_{L^\infty(\Omega)} \limsup_{n\to\infty} \frac{h_n^{\alpha_1} \hat{k}_n^{\beta_1}}{C_n^{p-1}} < \eta_1^{1-p}, \\ &\|a_2\|_{L^\infty(\Omega)} \limsup_{n\to\infty} \frac{k_n^{\alpha_2} \hat{h}_n^{\beta_2}}{C_n^{q-1}} < \eta_2^{1-q}, \end{aligned} \tag{S''}$$

where $\eta_1, \eta_2 \geq 1$ stem from estimates (3.20).

Remark 4.1. One can take $\gamma_1, \delta_1 \in [0, p-1]$ provided

$$\|a_1\|_{L^\infty(\Omega)} \limsup_{n\to\infty} \frac{h_n^{\alpha_1} \hat{k}_n^{\beta_1}}{C_n^{p-1}} + 2\|b_1\|_{L^\infty(\Omega)} < \eta_1^{1-p},$$

which implies the first inequality in (S''). A similar comment applies to γ_2, δ_2.

Theorem 4.2. *Let* (F_1'), (G_1'), *and* (S_1) *be satisfied. Then problem* (P) *admits a sequence of solutions* $\{(u_n,v_n)\}_n \subseteq C^1(\overline{\Omega})^2$ *such that* $(u_n,v_n) < (u_{n+1},v_{n+1})$ *for all* $n \in \mathbb{N}$. *Moreover,* $\lim_{n\to\infty} u_n = \lim_{n\to\infty} v_n = +\infty$ *uniformly in* $\overline{\Omega}$ *once* $h_n, \hat{h}_n \to +\infty$.

Proof. Define

$$K_n := C^1(\overline{\Omega})^2 \cap ([h_n,k_n] \times [\hat{h}_n,\hat{k}_n]),$$

as well as

$$D_n := \{w \in K_n : \|\nabla w\|_{L^\infty(\Omega)^2} \leq C_n\}.$$

If $(u, v, w) \in [h_n, k_n] \times [\hat{h}_n, \hat{k}_n] \times D_n$, then through ($\mathrm{F}_1'$) and (S″) we obtain

$$\begin{aligned} & |f(\cdot,u,v,\nabla w)| \\ & \quad \le \|a_1\|_{L^\infty(\Omega)} h_n^{\alpha_1} \hat{k}_n^{\beta_1} + \|b_1\|_{L^\infty(\Omega)} (C_n^{\gamma_1} + C_n^{\delta_1}) + \|c_1\|_{L^\infty(\Omega)} \\ & \quad \le \left(\frac{C_n}{\eta_1}\right)^{p-1} \end{aligned} \tag{4.1}$$

for any $n \in \mathbb{N}$ large enough. Likewise, (G_1') and (S″) yield

$$\begin{aligned} & |g(\cdot,u,v,\nabla w)| \\ & \quad \le \|a_2\|_{L^\infty(\Omega)} k_n^{\alpha_2} \hat{h}_n^{\beta_2} + \|b_2\|_{L^\infty(\Omega)} (C_n^{\gamma_2} + C_n^{\delta_2}) + \|c_2\|_{L^\infty(\Omega)} \\ & \quad \le \left(\frac{C_n}{\eta_2}\right)^{q-1}. \end{aligned} \tag{4.2}$$

Hence, from (3.20), with $K := K_n$, it follows $\Gamma(D_n) \subseteq D_n$, where Γ is given by (3.15). Let us point out that condition (3.21) was used in Theorem 3.8 only to achieve $\Gamma(D) \subseteq D$. Accordingly, here, it is unnecessary. Observe next that, thanks to (S′),

$$f(\cdot, k_n, v, \nabla w) \le 0 \le f(\cdot, h_n, v, \nabla w),$$
$$g(\cdot, u, \hat{k}_n, \nabla w) \le 0 \le g(\cdot, u, \hat{h}_n, \nabla w),$$

which easily force (3.3). So, hypothesis (H) of Theorem 3.8 is fulfilled. Thus, for every $n \in \mathbb{N}$, problem (P) possesses a solution $(u_n, v_n) \in K_n$. Since $k_n < h_{n+1}$ and $\hat{k}_n < \hat{h}_{n+1}$, we evidently have $(u_n, v_n) < (u_{n+1}, v_{n+1})$. Finally, if $h_n, \hat{h}_n \to +\infty$ then $\lim_{n\to\infty} u_n = \lim_{n\to\infty} v_n = +\infty$ uniformly in $\overline{\Omega}$. □

4.2. The super-linear case

The conditions below will be posited.

(F_2') There exist $\alpha_1 < 0 < \beta_1$, $\gamma_1, \delta_1 \in (p-1, +\infty)$, and $a_1, b_1 \in L^\infty(\Omega)$ such that

$$|f(x,s,t,\xi_1,\xi_2)| \le a_1(x) s^{\alpha_1} t^{\beta_1} + b_1(x)(|\xi_1|^{\gamma_1} + |\xi_2|^{\delta_1})$$

for all $(x, s, t, \xi_1, \xi_2) \in \Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N}$.

(G_2') There exist $\beta_2 < 0 < \alpha_2$, $\gamma_2, \delta_2 \in (q-1, +\infty)$, and $a_2, b_2 \in L^\infty(\Omega)$ such that

$$|g(x,s,t,\xi_1,\xi_2)| \le a_2(x) s^{\alpha_2} t^{\beta_2} + b_2(x)(|\xi_1|^{\gamma_2} + |\xi_2|^{\delta_2})$$

for all $(x, s, t, \xi_1, \xi_2) \in \Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N}$.

(S_2) There exist $\{h_n\}_n, \{\hat{h}_n\}_n, \{k_n\}_n, \{\hat{k}_n\}_n, \{C_n\}_n \subseteq (0, +\infty)$, with $C_n \to 0$, satisfying $k_{n+1} < h_n < k_n$, $\hat{k}_{n+1} < \hat{h}_n < \hat{k}_n$ and such that (S′)–(S″) are true for all $(x, s, t, \xi_1, \xi_2) \in \Omega \times [h_n, k_n] \times [\hat{h}_n, \hat{k}_n] \times B_{\mathbb{R}^N}(C_n)^2$, $n \in \mathbb{N}$.

Remark 4.1 can be adapted to (F_2')–(G_2').

Theorem 4.3. *Under assumptions* (F_2'), (G_2'), *and* (S_2), *problem* (P) *has a sequence of solutions* $\{(u_n, v_n)\}_n \subseteq C^1(\overline{\Omega})^2$ *such that* $(u_{n+1}, v_{n+1}) < (u_n, v_n)$ *for every* $n \in \mathbb{N}$. *Moreover,* $\lim_{n\to\infty} u_n = \lim_{n\to\infty} v_n = 0$ *uniformly in* $\overline{\Omega}$ *once* $k_n, \hat{k}_n \to 0$.

Proof. The argument is patterned after that of Theorem 4.2, because (4.1)–(4.2), written for $c_1 \equiv c_2 \equiv 0$, hold whenever $n \in \mathbb{N}$ is sufficiently large. □

Remark 4.4. Conditions (F_i') and (G_i'), $i = 1, 2$, above have been formulated on the whole $\Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N}$ just to avoid cumbersome statements. In fact, consider, e.g., Theorem 4.3 and suppose $k_n, \hat{k}_n \to 0$. Since C_n is arbitrarily small for n large, it suffices to request (F_2') in $\Omega \times (0, \delta]^2 \times B_{\mathbb{R}^N}(\delta)^2$ with appropriate $\delta > 0$, and the same arguments work. So, we can actually treat reactions f, g having any behavior far from the origin. A 'dual' comment holds for Theorem 4.2.

Example 4.5. Define, provided $(x, s, t, \xi_1, \xi_2) \in \Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N}$,

$$f(x, s, t, \xi_1, \xi_2) = \sin s + \frac{1}{2}\cos t, \quad g(x, s, t, \xi_1, \xi_2) = \frac{1}{2}\sin s + \cos t.$$

Inequalities (S'') are true because $a_i \equiv 0$, $i = 1, 2$. Choosing $h_n = \frac{\pi}{2} + 2\pi n$, $k_n = \frac{3}{2}\pi + 2\pi n$, $\hat{h}_n = 2\pi n$, $\hat{k}_n = \pi + 2\pi n$, and $C_n = n$, easily entails (S'). Hence, f and g comply with (S_1).

An example of nonlinearities, with both singular and convective terms, that fulfill (S_2) is the following.

Example 4.6. Set, for every $(x, s, t, \xi_1, \xi_2) \in \Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N}$,

$$\begin{aligned} f(x, s, t, \xi_1, \xi_2) &= \sin\frac{1}{s}\left(s^{\alpha_1}t^{\beta_1} - |\xi_1|^{\gamma_1} - |\xi_2|^{\delta_1}\right), \\ g(x, s, t, \xi_1, \xi_2) &= \cos\frac{1}{t}\left(s^{\alpha_2}t^{\beta_2} - |\xi_1|^{\gamma_2} - |\xi_2|^{\delta_2}\right), \end{aligned}$$

where

$$\min\{\gamma_1, \delta_1\} > \alpha_1 + \beta_1 > p - 1, \quad \min\{\gamma_2, \delta_2\} > \alpha_2 + \beta_2 > q - 1.$$

To check (S_2) one can pick $h_n = \left(\frac{\pi}{2} + 2\pi n\right)^{-1}$, $k_n = \left(-\frac{\pi}{2} + 2\pi n\right)^{-1}$, $\hat{h}_n = (2\pi + 2\pi n)^{-1}$, $\hat{k}_n = (\pi + 2\pi n)^{-1}$, and $C_n = \frac{1}{n}$.

Let us finally point out a simple consequence of Theorem 4.2.

Corollary 4.7. *Suppose* $h : \mathbb{R} \to \mathbb{R}$ *is continuous periodic and* $\alpha \in L^\infty(\Omega)$ *satisfies* $\inf_{\mathbb{R}} h \le \alpha \le \sup_{\mathbb{R}} h$. *Then the problem*

$$-\Delta_p u = h(u) - \alpha(x) \ \ in \ \ \Omega, \quad u > 0 \ \ in \ \ \Omega, \quad \frac{\partial u}{\partial \nu} = 0 \ \ on \ \ \partial\Omega$$

admits infinitely many solutions in $C^1(\overline{\Omega})$.

References

[1] U. Guarnotta, S.A. Marano, Infinitely many solutions to singular convective Neumann systems with arbitrarily growing reactions, J. Differ. Equ. 271 (2021) 849–863.