Corrigendum


U. Guarnotta, S.A. Marano *
Received 15 October 2020; accepted 1 November 2020

Abstract

We give a correct formulation of Theorems 4.2-4.3 in [1]. © 2020 Elsevier Inc. All rights reserved.

MSC: 35J57; 35J62; 35J75

Keywords: Neumann problem; Quasilinear elliptic system; Gradient dependence; Singular term; Arbitrary growth

Notation is the same as that adopted in [1]. Due to a technical mistake, the proofs of [1, Theorems 4.2-4.3] are incorrect. However, their conclusions still hold true provided a further condition (see (S1)–(S2) below) on the sign of nonlinearities is assumed. For the reader’s convenience, here, we give the amended version of the whole Section 4.

4. Infinitely many solutions

4.1. The sub-linear case

We make the hypotheses below.

DOI of original article: https://doi.org/10.1016/j.jde.2020.09.024.
* Corresponding author.
E-mail address: marano@dmi.unict.it (S.A. Marano).

https://doi.org/10.1016/j.jde.2020.11.015
0022-0396/© 2020 Elsevier Inc. All rights reserved.
(F') There exist $\alpha_1 < 0 < \beta_1$, $\gamma_1, \delta_1 \in [0, p - 1)$, and $a_1, b_1, c_1 \in L^\infty(\Omega)$ such that

$$|f(x, s, t, \xi_1, \xi_2)| \leq a_1(x)s^{\alpha_1}t^{\beta_1} + b_1(x)(|\xi_1|^{\gamma_1} + |\xi_2|^{\delta_1}) + c_1(x)$$

for all $(x, s, t, \xi_1, \xi_2) \in \Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N}$.

(G') There exist $\beta_2 < 0 < \alpha_2$, $\gamma_2, \delta_2 \in [0, q - 1)$, and $a_2, b_2, c_2 \in L^\infty(\Omega)$ such that

$$|g(x, s, t, \xi_1, \xi_2)| \leq a_2(x)s^{\alpha_2}t^{\beta_2} + b_2(x)(|\xi_1|^{\gamma_2} + |\xi_2|^{\delta_2}) + c_2(x)$$

for all $(x, s, t, \xi_1, \xi_2) \in \Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N}$.

(S) There exist $\{h_n\}_n, \{\hat{h}_n\}_n, \{k_n\}_n, \{C_n\}_n \subseteq (0, +\infty)$, with $C_n \to +\infty$, satisfying $h_n < k_n < h_{n+1}, \hat{h}_n < \hat{k}_n < \hat{h}_{n+1}$, and

$$f(x, k_n, t, \xi_1, \xi_2) \leq 0 \leq f(x, h_n, t, \xi_1, \xi_2),$$
$$g(x, s, \hat{k}_n, \xi_1, \xi_2) \leq 0 \leq g(x, s, \hat{h}_n, \xi_1, \xi_2)$$

for all $(x, s, t, \xi_1, \xi_2) \in \Omega \times [h_n, k_n] \times [\hat{h}_n, \hat{k}_n] \times B_{\mathbb{R}^{2N}}(C_n)^2, n \in \mathbb{N}$. Further,

$$\|a_1\|_{L^\infty(\Omega)} \limsup_{n \to \infty} \frac{h_n^{\alpha_1} \hat{k}_n^{\beta_1}}{C_n^{p-1}} < \eta_1^{1-p}$$
$$\|a_2\|_{L^\infty(\Omega)} \limsup_{n \to \infty} \frac{k_n^{\alpha_2} \hat{h}_n^{\beta_2}}{C_n^{q-1}} < \eta_2^{1-q}$$

where $\eta_1, \eta_2 \geq 1$ stem from estimates (3.20).

**Remark 4.1.** One can take $\gamma_1, \delta_1 \in [0, p - 1]$ provided

$$\|a_1\|_{L^\infty(\Omega)} \limsup_{n \to \infty} \frac{h_n^{\alpha_1} \hat{k}_n^{\beta_1}}{C_n^{p-1}} + 2\|b_1\|_{L^\infty(\Omega)} < \eta_1^{1-p},$$

which implies the first inequality in $(S'')$. A similar comment applies to $\gamma_2, \delta_2$.

**Theorem 4.2.** Let $(F')$, $(G')$, and $(S)$ be satisfied. Then problem (P) admits a sequence of solutions $\{(u_n, v_n)\}_n \subseteq C^1(\overline{\Omega})^2$ such that $(u_n, v_n) < (u_{n+1}, v_{n+1})$ for all $n \in \mathbb{N}$. Moreover, $\lim_{n \to \infty} u_n = \lim_{n \to \infty} v_n = +\infty$ uniformly in $\overline{\Omega}$ once $h_n, \hat{h}_n \to +\infty$.

**Proof.** Define

$$K_n := C^1(\overline{\Omega})^2 \cap ([h_n, k_n] \times [\hat{h}_n, \hat{k}_n])$$

as well as

$$D_n := \{w \in K_n : \|\nabla w\|_{L^\infty(\Omega)^2} \leq C_n\}.$$
If \((u, v, w) \in [h_n, k_n] \times [\hat{h}_n, \hat{k}_n] \times D_n\), then through \((F')\) and \((S'')\) we obtain

\[
|f(\cdot, u, v, \nabla w)| \\
\leq ||a_1||_{L^\infty(\Omega)}h_n^{\alpha_1}k_n^{\beta_1} + ||b_1||_{L^\infty(\Omega)}(C_n^{\gamma_1} + C_n^{\delta_1}) + ||c_1||_{L^\infty(\Omega)} \\
\leq \left( \frac{C_n}{\eta_1} \right)^{p-1}
\]

(4.1)

for any \(n \in \mathbb{N}\) large enough. Likewise, \((G'_1)\) and \((S'')\) yield

\[
|g(\cdot, u, v, \nabla w)| \\
\leq ||a_2||_{L^\infty(\Omega)}k_n^{\alpha_2}\hat{h}_n^{\beta_2} + ||b_2||_{L^\infty(\Omega)}(C_n^{\gamma_2} + C_n^{\delta_2}) + ||c_2||_{L^\infty(\Omega)} \\
\leq \left( \frac{C_n}{\eta_2} \right)^{q-1}
\]

(4.2)

Hence, from (3.20), with \(K := K_n\), it follows \(\Gamma(D_n) \subseteq D_n\), where \(\Gamma\) is given by (3.15). Let us point out that condition (3.21) was used in Theorem 3.8 only to achieve \(\Gamma(D) \subseteq D\). Accordingly, here, it is unnecessary. Observe next that, thanks to \((S')\),

\[
f(\cdot, k_n, v, \nabla w) \leq 0 \leq f(\cdot, h_n, v, \nabla w),
\]

\[
g(\cdot, u, \hat{k}_n, \nabla w) \leq 0 \leq g(\cdot, u, \hat{h}_n, \nabla w),
\]

which easily force (3.3). So, hypothesis (H) of Theorem 3.8 is fulfilled. Thus, for every \(n \in \mathbb{N}\), problem \((P)\) possesses a solution \((u_n, v_n) \in K_n\). Since \(k_n < h_{n+1}\) and \(\hat{k}_n < \hat{h}_{n+1}\), we evidently have \((u_n, v_n) < (u_{n+1}, v_{n+1})\). Finally, if \(h_n, \hat{h}_n \to +\infty\) then \(\lim_{n \to \infty} u_n = \lim_{n \to \infty} v_n = +\infty\) uniformly in \(\Omega \). \(\square\)

4.2. The super-linear case

The conditions below will be posited.

\((F'_2)\) There exist \(\alpha_1 < 0 < \beta_1, \gamma_1, \delta_1 \in (p - 1, +\infty)\), and \(a_1, b_1 \in L^\infty(\Omega)\) such that

\[
|f(x, s, t, \xi_1, \xi_2)| \leq a_1(x)s^{\alpha_1}t^{\beta_1} + b_1(x)(|\xi_1|^{\gamma_1} + |\xi_2|^{\delta_1})
\]

for all \((x, s, t, \xi_1, \xi_2) \in \Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N}\).

\((G'_2)\) There exist \(\beta_2 < 0 < \alpha_2, \gamma_2, \delta_2 \in (q - 1, +\infty)\), and \(a_2, b_2 \in L^\infty(\Omega)\) such that

\[
|g(x, s, t, \xi_1, \xi_2)| \leq a_2(x)s^{\alpha_2}t^{\beta_2} + b_2(x)(|\xi_1|^{\gamma_2} + |\xi_2|^{\delta_2})
\]

for all \((x, s, t, \xi_1, \xi_2) \in \Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N}\).

\((S'_2)\) There exist \((h_n)_n, (\hat{h}_n)_n, (k_n)_n, (\hat{k}_n)_n, (C_n)_n \subseteq (0, +\infty)\), with \(C_n \to 0\), satisfying \(k_{n+1} < h_n < k_n, \hat{k}_{n+1} < \hat{h}_n < \hat{k}_n\) and such that \((S')-(S'')\) are true for all \((x, s, t, \xi_1, \xi_2) \in \Omega \times [h_n, k_n] \times [\hat{h}_n, \hat{k}_n] \times B_{\mathbb{R}^N}(C_n)^2, n \in \mathbb{N}\).
Remark 4.1 can be adapted to (F’\_2)–(G’\_2).

**Theorem 4.3.** Under assumptions (F’\_2), (G’\_2), and (S\_2), problem (P) has a sequence of solutions \{(u\_n, v\_n)\}_{n} \subseteq C^1(\overline{\Omega})^2 such that (u\_{n+1}, v\_{n+1}) < (u\_n, v\_n) for every \( n \in \mathbb{N} \). Moreover, \( \lim_{n \to \infty} u\_n = \lim_{n \to \infty} v\_n = 0 \) uniformly in \( \overline{\Omega} \) once \( k\_n, \hat{k}\_n \to 0 \).

**Proof.** The argument is patterned after that of Theorem 4.2, because (4.1)–(4.2), written for \( c_1 = c_2 = 0 \), hold whenever \( n \in \mathbb{N} \) is sufficiently large. □

**Remark 4.4.** Conditions (F’\_i) and (G’\_i), \( i = 1, 2 \), above have been formulated on the whole \( \Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N} \) just to avoid cumbersome statements. In fact, consider, e.g., Theorem 4.3 and suppose \( k\_n, \hat{k}\_n \to 0 \). Since \( C\_n \) is arbitrarily small for \( n \) large, it suffices to request (F’\_2) in \( \Omega \times (0, \delta)^2 \times B_{\mathbb{R}^N}(\delta)^2 \) with appropriate \( \delta > 0 \), and the same arguments work. So, we can actually treat reactions \( f, g \) having any behavior far from the origin. A ‘dual’ comment holds for Theorem 4.2.

**Example 4.5.** Define, provided \((x, s, t, \xi_1, \xi_2) \in \Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N},

\[
f(x, s, t, \xi_1, \xi_2) = \sin s + \frac{1}{2} \cos t, \quad g(x, s, t, \xi_1, \xi_2) = \frac{1}{2} \sin s + \cos t.
\]

Inequalities (S’\_\text{''}) are true because \( a_i \equiv 0, i = 1, 2 \). Choosing \( h\_n = \frac{\pi}{2} + 2\pi n, \ k\_n = \frac{3}{2} \pi + 2\pi n, \ \hat{h}\_n = 2\pi n, \ \hat{k}\_n = \pi + 2\pi n, \) and \( C\_n = n \), easily entails (S’). Hence, \( f \) and \( g \) comply with (S\_1).

An example of nonlinearities, with both singular and convective terms, that fulfill (S\_2) is the following.

**Example 4.6.** Set, for every \((x, s, t, \xi_1, \xi_2) \in \Omega \times (0, +\infty)^2 \times \mathbb{R}^{2N},

\[
f(x, s, t, \xi_1, \xi_2) = \sin \frac{1}{s} (s^{\alpha_1} t^{\beta_1} - |\xi_1|^{\gamma_1} - |\xi_2|^{\beta_1}) ,
\]

\[
g(x, s, t, \xi_1, \xi_2) = \cos \frac{1}{t} (s^{\alpha_2} t^{\beta_2} - |\xi_1|^{\gamma_2} - |\xi_2|^{\beta_2}) ,
\]

where

\[
\min\{\gamma_1, \delta_1\} > \alpha_1 + \beta_1 > p - 1, \quad \min\{\gamma_2, \delta_2\} > \alpha_2 + \beta_2 > q - 1.
\]

To check (S\_2) one can pick \( h\_n = \left( \frac{\pi}{2} + 2\pi n \right)^{-1} , \ k\_n = \left( -\frac{\pi}{2} + 2\pi n \right)^{-1} , \ \hat{h}\_n = \left( 2\pi + 2\pi n \right)^{-1} , \ \hat{k}\_n = \left( \pi + 2\pi n \right)^{-1} , \) and \( C\_n = \frac{1}{n} \).

Let us finally point out a simple consequence of Theorem 4.2.

**Corollary 4.7.** Suppose \( h : \mathbb{R} \to \mathbb{R} \) is continuous periodic and \( \alpha \in L^\infty(\Omega) \) satisfies \( \inf_{\mathbb{R}} h \leq \alpha \leq \sup_{\mathbb{R}} h \). Then the problem
\[-\Delta_p u = h(u) - \alpha(x) \text{ in } \Omega, \quad u > 0 \text{ in } \Omega, \quad \frac{\partial u}{\partial \nu} = 0 \text{ on } \partial \Omega\]

admits infinitely many solutions in $C^1(\overline{\Omega})$.

References