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Optical solitons and other solutions to Kaup–Newell equation with Jacobi elliptic function expansion method

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Abstract
In this paper, we studied Kaup–Newell (KN) equation in coupled vector form without four-wave mixing terms in birefringent fibers. We employed Jacobi elliptic function expansion method in order to demonstrate sub-pico-second optical soliton solutions. Beside bright and dark solitons, Jacobi elliptic function solutions and hyperbolic solutions are obtained. Moreover, the graphs for some solution are presented.

Keywords Jacobi elliptic function expansion method · Kaup–Newell model · Jacobi elliptic function solutions · Hyperbolic function solutions · Dark solitons · Bright solitons

1 Introduction

In recent years, exact solutions to nonlinear partial differential equations (NLPDEs) have played an important role in the study of many phenomena, particularly nonlinear physical phenomena such as hydrodynamics, fluid mechanics, plasma physics, optics, solid state physics and also in various fields of the engineering and science, it also gave researchers an idea of understanding many physical phenomena. Optical solitons form the basic fabric in the field of telecommunication industry. They are the carriers for the transfer of information through optical fibers. These information transmission carriers serve the modern-day telecommunication system through Internet, which include

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electronic mail transmission, social media activities. The captivating technology of sub-pico second pulses that propagate through optical fibers is modeled with Kaup–Newell's (KN) equation (see [1–10]). The KNE is one of the three forms of derivative nonlinear Schrodinger's equations that govern this soliton dynamics. In this paper, we investigate Kaup–Newell equation in coupled vector form without four-wave mixing terms in birefringent fibers. We employ the Jacobi elliptic function expansion method in order to demonstrate sub-pico-second optical soliton solutions. Moreover, we obtain bright and dark solitons. Further, hyperbolic and Jacobi elliptic functions solutions are also reported.

2 A description of the Jacobi elliptic function expansion method

We assume that the general form of the nonlinear partial differential equation (NLPDE) in the form:

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \ldots) = 0, \tag{1}$$

P is a polynomial function in $u(x,t)$ and its different partial derivatives.
Main steps of the Jacobi elliptic function expansion method are as follows (see [11–13]):

*Step*1: We assume that the traveling wave solution of the Eq. (1) takes the form:

$$u(x,t) = u(\xi), \qquad \xi = kx - \rho\, t, \tag{2}$$

k and ρ are real constants.
Substituting (2) in (1), we obtain the nonlinear ordinary differential Eq. (NLODE)in the form:

$$F(u, u', u'', \ldots) = 0, \tag{3}$$

where F is a polynomial of $u(\xi)$ and its total derivatives $u', u'', \ldots,$ where the prime $'$ denotes $\frac{d}{d\xi}$.

*Step*2: We assume that the solution of the Eq.(3) takes the form:

$$u(\xi) = g_0 + \sum_{i=1}^{N} \left(\frac{z(\xi)}{1+z(\xi)^2} \right)^{i-1} \left(g_i \frac{z(\xi)}{1+z(\xi)^2} + f_i \frac{1-z(\xi)^2}{1+z(\xi)^2} \right), \tag{4}$$

where g_i and f_i are constants, such that $g_N \neq 0$ or $f_N \neq 0$, and N is determined by balancing the linear term of the highest order derivative and nonlinear terms, while $z(\xi)$ can be determined by the first kind of Jacobi Elliptic equation:

$$\left(\frac{dz(\xi)}{d\xi} \right)^2 = s + c\, z^2(\xi) + r\, z^4(\xi), \tag{5}$$

s, c and r are constants.

Table 1 They are all Jacobi elliptic functions

No.	s	c	r	$z(\xi)$
1	1	$-(1+m^2)$	m^2	$\operatorname{sn}\xi$
2	$1-m^2$	$2m^2-1$	$-m^2$	$\operatorname{cn}\xi$
3	m^2	$-(1+m^2)$	1	$\operatorname{ns}\xi=(\operatorname{sn}\xi)^{-1}$
4	$-m^2$	$2m^2-1$	$1-m^2$	$\operatorname{nc}\xi=(\operatorname{cn}\xi)^{-1}$
5	$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$	$\operatorname{ns}\xi\pm\operatorname{cs}\xi$
6	$\frac{1-m^2}{4}$	$\frac{1+2m^2}{2}$	$\frac{1-m^2}{4}$	$\operatorname{nc}\xi\pm\operatorname{sc}\xi$ *or* $\frac{\operatorname{cn}\xi}{1\pm\operatorname{sn}\xi}$

*Step*3: Determine the positive integer N in (4) by balancing the highest order derivatives and the nonlinear term in Eq. (3).

*Step*4: Substituting (4) along with Eq. (5) into (3) and collecting all the coefficients of $z^i(\xi)(i=0,1,2,\ldots)$ then setting these coefficients to zero, yield a set of algebraic equations, which can be solved by using the Maple or Mathematica to find the values of $f_i, g_i, s, c, r, \rho, k$.

*Step*5: It is well-known that Eq. (5) has families of Jacobi elliptic function solutions as follows (Table 1).

*Step*6: Substituting the values of $g_0,\ g_i,\ f_i,\ s,\ c,\ r,\ z(\xi),\ \rho,\ k$, as well as, the solutions of Eq. (5) obtained in Step 5, into (4) we have the Jacobian elliptic functions solution with the optical solitons solutions of the Eq. (1).

2.1 Some of the properties of Jacobian elliptic functions

The three Jacobi elliptic functions $\operatorname{sn}(\xi|m),\ \operatorname{cn}(\xi|m),\ \operatorname{dn}(\xi|m)$ are functions of the variable ξ and the elliptic modulus parameter $m=k^2,\quad 0\le m\le 1$. The inverse functions are most easily defined in terms of elliptic integrals. For example

$$\operatorname{sn}^{-1}(\xi|m)=\int_0^x \frac{dz}{\sqrt{(}1-z^2)(1-mz^2)},$$

with similar relation $\operatorname{cn}^{-1}(\xi|m)$ and $\operatorname{dn}^{-1}(\xi|m)$.
The Jacobi elliptic functions $\operatorname{sn}\xi=\operatorname{sn}(\xi|m)$, $\operatorname{cn}\xi=\operatorname{cn}(\xi|m)$ and $\operatorname{dn}\xi=\operatorname{dn}(\xi|m)$ are double periodic and possess properties of trigonometric functions $\operatorname{sn}^2\xi+\operatorname{cn}^2\xi=1,\ \operatorname{dn}^2\xi=1-m^2\operatorname{sn}^2\xi$.
If $m\to 1$, the Jacobi functions degenerate to the hyperbolic functions $\operatorname{sn}\xi\to\tanh\xi,\ \operatorname{cn}\xi\to\operatorname{sech}\xi,\ \operatorname{dn}\xi\to\operatorname{sech}\xi,\ \operatorname{cs}\xi\to\operatorname{csch}\xi$ and $\xi\to\operatorname{csch}\xi$, but when $m\to 0$, the Jacobi functions degenerate to the trigonometric functions $\operatorname{sn}\xi\to\sin\xi,\ \operatorname{cn}\xi\to\cos\xi,\ \operatorname{dn}\xi\to 1,\ \operatorname{cs}\xi\to\cot\xi$ and $\xi\to\csc\xi$.

3 Kaup–Newell model

The Kaup–Newell (KN) model in polarization-preserving fibers (see [14–24]) represented by:

$$q_t + i\, a\, q_{xx} + b\, (\, |q|^2\, q\,)_x = 0, \tag{6}$$

where a is a coefficient of GVD, and b assures the existence of the nonlinearity. The GVD is used to show how nonlinearity will affect the sub pico–second optical pulse traveling through it. Finally, this model is impossible without non-linearity, because the sub pico- second pulses only occur when the delicate balance of nonlinear and GVD is established.

The KN system in the form of coupled vector without FWM reads

$$\psi_t + i\, a_1\, \psi_{xx} + \gamma_1\, \left(|\phi|^2\, \phi\right)_x + \lambda_1\, \left(|\psi|^2\, \psi\right)_x = 0, \tag{7}$$

$$\phi_t + i\, a_2\, \phi_{xx} + \gamma_2\, \left(|\psi|^2\, \psi\right)_x + \lambda_2\, \left(|\phi|^2\, \phi\right)_x = 0, \tag{8}$$

with the constants a_i and $\lambda_i,\ \gamma_i$ that assure the existence of the GVD and nonlinearity sequentially.

4 Application on the governing model

We assume that the solutions of Eqs. (7) and (8) have the form:

$$\psi(x, t) = P_1(\vartheta)\, e^{i\, \varphi(x,t)}, \tag{9}$$

$$\phi(x, t) = P_2(\vartheta)\, e^{i\, \varphi(x,t)}, \tag{10}$$

where

$$\vartheta = x - \rho t, \tag{11}$$

and

$$\varphi(x, t) = -\kappa\, x + \omega\, t + \zeta, \tag{12}$$

where $P_j(\vartheta),\ \ j = 1, 2, \varphi(x, t)\,, \rho, \kappa, \omega$ and ζ that represent the amplitude component, phase function, speed of the soliton, frequency, wave number and phase respectively. Substituting Eqs. (9) and (10) into Eqs.(7) and (8), and separate the real and imaginary parts, we get

$$-a_j\, P_j'' + \left(a_j\, \kappa^2 - \omega\right)\, P_j + \kappa\, \lambda_j\, P_j^3 + \kappa\, \gamma_j\, P_{\tilde{j}}^3 = 0, \tag{13}$$

$$\left(2\, a_j\, \kappa - \rho\right)\, P_j' + 3\, \lambda_j\, P_j^2\, P_j' + 3\, \gamma_j\, P_{\tilde{j}}^2\, P_{\tilde{j}}' = 0, \tag{14}$$

where $j = 1, 2$ and $\tilde{j} = 3 - j$. Using a balance rule leads to

$$P_j = P_{\tilde{j}}, \tag{15}$$

From the Eq. (14) then, the speed of the soliton is:

$$\rho = 2\, a_j\, \kappa + \left(\gamma_j + \lambda_j\right)\; P_j^2, \tag{16}$$

and

$$a_j\; P_j'' - \kappa\; \left(\gamma_j + \lambda_j\right)\; P_j^3 + \left(\omega - a_j\, \kappa^2\right)\; P_j = 0. \tag{17}$$

Therefore, the equation required for the exact solution of Eqs. (7) and (8) is represented by (17). From step 2 in Sect. 2 and by applying the principle of the balance criteria between the terms of P_j^3 and P_j'' in Eq. (17), we get N =1, then, the solution of Eq. (17) reads

$$P(\vartheta) = g_0 + g_1\, \left(\frac{z(\vartheta)}{1 + z(\vartheta)^2}\right) + f_1 \left(\frac{1 - z(\vartheta)^2}{1 + z(\vartheta)^2}\right), \tag{18}$$

where $g_i,\ i = 0, 1$ and f_1 are the constants, so that $g_1 \neq 0$ or $f_1 \neq 0$. Substituting Eq. (18) along with Eq. (5) into Eq. (17), and collecting all the coefficients of $z^i(\xi)$, $(i = 0, 1, \ldots, 6)$, and by setting the result to zero, we obtain the following system:

z^0 coeff.:

$$\begin{aligned}
&- f_1^2\, \kappa\; \left(\gamma_j + \lambda_j\right)(3\, g_0 + 1) + f_1\, \left(-3\, g_0^2\, \kappa\; \left(\gamma_j + \lambda_j\right) - a_j\; \left(\kappa^2 + 4\, s\right) + \omega\right)\\
&\quad - g_0\, (g_0^2\, \kappa\; \left(\gamma_j + \lambda_j\right) +\\
&\quad \kappa^2\, a_j - \omega) = 0,
\end{aligned}$$

z coeff.:

$$\begin{aligned}
&g_1\, \left(a_j\; \left(c - \kappa^2 - 6s\right)\right.\\
&\quad \left. -6 f_1\, g_0 \kappa\; \left(\gamma_j + \lambda_j\right) - 3\, f_1^2\, \kappa\; \left(\gamma_j + \lambda_j\right) - 3 g_0^2\, \kappa\, \gamma_j - 3\, g_0^2\, \kappa\, \lambda_j + \omega\right) = 0,
\end{aligned}$$

z^2 coeff.:

$$\begin{aligned}
&f_1\, \left(-a_j\; \left(8\, c + \kappa^2 - 12\, s\right) - 3\, g_0^2\, \kappa\; \left(\gamma_j + \lambda_j\right) - 3\, g_1^2\, \kappa\, \gamma_j - 3\, g_1^2 \kappa\, \lambda_j + \omega\right)\\
&\quad + 3\, f_1^2\, g_0\, \kappa \left(\gamma_j + \lambda_j\right) + 3\, f_1^3\, \kappa\; \left(\gamma_j + \lambda_j\right)\\
&\quad - 3\, g_0\, \left(g_0^2\, \kappa\; \left(\gamma_j + \lambda_j\right) + g_1^2\, \kappa\, \gamma_j + g_1^2\, \kappa\, \lambda_j + \kappa^2\, a_j - \omega\right) = 0,
\end{aligned}$$

z^3 coeff.:

$$g_1\ (2a_j(3\ c+\kappa^2-r-s)-6\ f_1^2\ \kappa\ (\gamma_j+\lambda_j)+6\ g_0^2\ \kappa\ \gamma_j+g_1^2\ \kappa\ \gamma_j$$
$$+6\ g_0^2\ \kappa\ \lambda_j+g_1^2\ \kappa\ \lambda_j-2\omega)=0,$$

z^4 coeff.:

$$f_1\ \left(a_j\left(8\ c+\kappa^2-12\ r\right)+3\ g_0^2\kappa\ \left(\gamma_j+\lambda_j\right)+3\ g_1^2\ \kappa\ \gamma_j+3\ g_1^2\ \kappa\ \lambda_j-\omega\right)$$
$$+3\ f_1^2\ g_0\ \kappa\ \left(\gamma_j+\lambda_j\right)-3\ f_1^3\ \kappa\left(\gamma_j+\lambda_j\right)$$
$$-3\ g_0\left(g_0^2\ \kappa\ \left(\gamma_j+\lambda_j\right)+g_1^2\ \kappa\ \gamma_j+g_1^2\ \kappa\ \lambda_j+\kappa^2\ a_j-\omega\right)=0,$$

z^5 coeff.:

$$g_1\left(a_j\ \left(c-\kappa^2-6\ r\right)+6\ f_1 g_0\kappa\left(\gamma_j+\lambda_j\right)\right.$$
$$\left.-3\ f_1^2\kappa\left(\gamma_j+\lambda_j\right)-3\ g_0^2\ \kappa\ \gamma_j-3\ g_0^2\ \kappa\ \lambda_j+\omega\right)=0,$$

z^6 coeff.:

$$-3\ f_1^2\ g_0\ \kappa\ \left(\gamma_j+\lambda_j\right)+f_1^3\ \kappa\ \left(\gamma_j+\lambda_j\right)-g_0\ \left(g_0^2\ \kappa\ \left(\gamma_j+\lambda_j\right)+\kappa^2\ a_j-\omega\right)$$
$$+f_1\ (3\ g_0^2\ \kappa\left(\gamma_j+\lambda_j\right)+a_j\left(\kappa^2+4\ r\right)-\omega)=0.$$

This system of solutions is obtained by using Mathematica. Consequently, we obtain six types of solutions as follows:

Type 1 When $s=1$, $c=-(1+m^2)$, $r=m^2$ and $z(\vartheta)=\mathrm{sn}(\vartheta)$. Through $\mathrm{sn}(\vartheta,1)\rightarrow\tanh(\vartheta)$,then, we have two results

Result 1

$$g_0=0,\quad g_1=\pm\frac{4\sqrt{2\ \omega}}{\sqrt{\kappa\ \left(\kappa^{\ 2}+8\right)\ \left(\gamma_j+\lambda_j\right)}},\quad f_1=0,$$
$$m=1,\quad a_j=\frac{\omega}{\kappa^{\ 2}+8}$$

Then, the dark wave solutions of Eqs. (7) and (8) are:

$$\psi(x,t)=\pm\frac{2\sqrt{2\ \omega}}{\sqrt{\kappa\ \left(\kappa^2+8\right)\ \left(\gamma_1+\lambda_1\right)}}\ \tanh\ [2\ (x-\rho t)]\ e^{i\ (-\kappa\ x\ +\ \omega\ t+\zeta\ )},\tag{19}$$

$$\phi(x,t)=\pm\frac{2\sqrt{2\ \omega}}{\sqrt{\kappa\ \left(\kappa^2+8\right)\ \left(\gamma_2+\lambda_2\right)}}\ \tanh\ [2\ (x-\rho t)]\ e^{i\ (-\kappa\ x\ +\ \omega\ t+\zeta\ )},\tag{20}$$

and $\kappa\,\omega\,\left(\gamma_j+\lambda_j\right)>0\,,\ j=1,\ 2.$

Result 2

$$g_0=0,\quad g_1=0,\quad f_1=\pm\frac{2\sqrt{2\,\omega}}{\sqrt{-\kappa\,\left(\kappa^{\,2}-4\right)\left(\gamma_j+\lambda_j\right)}},\quad m=1,\quad a_j=\frac{\omega}{\kappa^2-4}$$

Then, the bright wave solutions of Eqs. (7) and (8) are:

$$\psi(x,t)=\pm\frac{2\sqrt{2\,\omega}}{\sqrt{-\kappa\,\left(\kappa^2-4\right)\,\left(\gamma_1+\lambda_1\right)}}\ \operatorname{sech}\,[2\,(x-\rho t)]\ e^{i\,(-\kappa\,x\,+\,\omega\,t+\zeta\,)},\tag{21}$$

$$\phi(x,t)=\pm\frac{2\sqrt{2\,\omega}}{\sqrt{-\kappa\,\left(\kappa^2-4\right)\,\left(\gamma_2+\lambda_2\right)}}\ \operatorname{sech}\,[2\,(x-\rho t)]\ e^{i\,(-\kappa\,x\,+\,\omega\,t+\zeta\,)},\tag{22}$$

and $\kappa\,\omega\,\left(\kappa^2-4\right)\left(\gamma_j+\lambda_j\right)<0\,,\ j=1,\ 2.$

Type 2 When $s=1-m^2\,,\ c=2\,m^2-1,\ r=-m^2$ and $z(\vartheta)=\operatorname{cn}(\vartheta)$, we have two results:

Result 1

$$g_0=\pm\frac{\sqrt{\omega}}{\sqrt{-\kappa\left(2\kappa^2-5\right)\left(\gamma_j+\lambda_j\right)}},\ g_1=0,\ f_1=\pm\frac{2\sqrt{\omega}}{\sqrt{-\kappa\left(2\kappa^2-5\right)\left(\gamma_j+\lambda_j\right)}},$$
$$m=\frac{1}{2},\ a_j=\frac{2\omega}{2\kappa^2-5}$$

Then, the Jacobi elliptic function solutions of the Eqs. (7) and (8) are:

$$\psi(x,t)=\pm\frac{\sqrt{\omega}}{\sqrt{-\kappa\,\left(2\,\kappa^2-5\right)\,\left(\gamma_1+\lambda_1\right)}}\ \left(\frac{3-\operatorname{cn}[x-\rho t]^2}{1+\operatorname{cn}[x-\rho t]^2}\right)\ e^{i\,(-\kappa\,x\,+\,\omega\,t+\zeta\,)},\tag{23}$$

$$\phi(x,t)=\pm\frac{\sqrt{\omega}}{\sqrt{-\kappa\,\left(2\,\kappa^2-5\right)\,\left(\gamma_2+\lambda_2\right)}}\ \left(\frac{3-\operatorname{cn}[x-\rho t]^2}{1+\operatorname{cn}[x-\rho t]^2}\right)\ e^{i\,(-\kappa\,x\,+\,\omega\,t+\zeta\,)},\tag{24}$$

and $\kappa\,\omega\,\left(2\,\kappa^2-5\right)\left(\gamma_j+\lambda_j\right)<0\,,\ j=1,\ 2.$

Result 2

$$g_0 = \pm\frac{\sqrt{\omega}}{\sqrt{\kappa\left(2\kappa^2+5\right)\left(\gamma_j+\lambda_j\right)}},\, g_1 = 0,\, f_1 = \mp\frac{2\sqrt{\omega}}{\sqrt{\kappa\left(2\kappa^2+5\right)\left(\gamma_j+\lambda_j\right)}},$$
$$m = \frac{\sqrt{3}}{2},\, a_j = \frac{2\omega}{2\kappa^2+5}$$

Then, the Jacobi elliptic function solutions of the Eqs. (7) and (8) are:

$$\psi(x,t) = \pm\frac{\sqrt{\omega}}{\sqrt{\kappa\ \left(2\,\kappa^2+5\right)\ \left(\gamma_1+\lambda_1\right)}}\ \left(\frac{-1+3\,\mathrm{cn}[x-\rho t]^2}{1+\mathrm{cn}[x-\rho t]^2}\right) e^{i\ (-\kappa\ x\ +\ \omega\ t+\zeta\ )}, \tag{25}$$

$$\phi(x,t) = \pm\frac{\sqrt{\omega}}{\sqrt{\kappa\ \left(2\,\kappa^2+5\right)\ \left(\gamma_2+\lambda_2\right)}}\ \left(\frac{-1+3\,\mathrm{cn}[x-\rho t]^2}{1+\mathrm{cn}[x-\rho t]^2}\right) e^{i\ (-\kappa\ x\ +\ \omega\ t+\zeta\ )}, \tag{26}$$

and $\kappa\ \omega\ \left(\gamma_j+\lambda_j\right) > 0\ ,\ \ j = 1,\ 2.$

Type 3 When $s = m^2\ ,\ c = -(\ m^2+1),\ r = 1$ and $z(\vartheta) = \mathrm{ns}(\vartheta)$. Through $\mathrm{ns}(\vartheta, 1) \rightarrow \coth(\vartheta)$, then, we have two results

Result 1

$$g_0 = 0,\quad g_1 = \pm\frac{4\,\sqrt{2\,\omega}}{\sqrt{\kappa\ \left(\kappa^2+8\right)\ \left(\gamma_j+\lambda_j\right)}},\quad f_1 = 0,\quad m = 1,\quad a_j = \frac{\omega}{\kappa^2+8}.$$

Then, the dark wave solutions of the Eqs. (7) and (8) are:

$$\psi(x,t) = \pm\frac{2\,\sqrt{2\,\omega}}{\sqrt{\kappa\ \left(\kappa^2+8\right)\ \left(\gamma_1+\lambda_1\right)}}\ \tanh[2\ (x-\rho\ t)]\ e^{i\ (-\kappa\ x\ +\ \omega\ t+\zeta\ )}, \tag{27}$$

$$\phi(x,t) = \pm\frac{2\,\sqrt{2\,\omega}}{\sqrt{\kappa\ \left(\kappa^2+8\right)\ \left(\gamma_2+\lambda_2\right)}}\ \tanh[2\ (x-\rho\ t)]\ e^{i\ (-\kappa\ x\ +\ \omega\ t+\zeta\ )}, \tag{28}$$

and $\kappa\ \omega\ \left(\gamma_j+\lambda_j\right) > 0\ ,\ \ j = 1,\ 2.$

Result 2

$$g_0 = 0,\quad g_1 = 0,\quad f_1 = \pm\frac{2\,\sqrt{2\,\omega}}{\sqrt{-\kappa\ \left(\kappa^2-4\right)\ \left(\gamma_j+\lambda_j\right)}},\quad m = 1,\quad a_j = \frac{\omega}{\kappa^2-4}$$

Then, the bright wave solutions of the Eqs. (7) and (8) are:

$$\psi(x,t) = \mp \frac{2\sqrt{2\,\omega}}{\sqrt{-\kappa\,\left(\kappa^2-4\right)\,(\gamma_1+\lambda_1)}}\ \operatorname{sech}[2\,(x-\rho\,t)]\ e^{i\,(-\kappa\,x\,+\,\omega\,t+\zeta\,)}, \quad (29)$$

$$\phi(x,t) = \mp \frac{2\sqrt{2\,\omega}}{\sqrt{-\kappa\,\left(\kappa^2-4\right)\,(\gamma_2+\lambda_2)}}\ \operatorname{sech}[2\,(x-\rho\,t)]\ e^{i\,(-\kappa\,x\,+\,\omega\,t+\zeta\,)}, \quad (30)$$

and $\kappa\,\omega\,(\kappa^2-4)\,\left(\gamma_j+\lambda_j\right)<0\,,\ j=1,\ 2.$

Type 4 When $s=-m^2\,,\ c=2\,m^2-1,\ r=1-m^2$ and $z(\vartheta)=\operatorname{nc}(\vartheta)$, we have two results:

Result 1

$$g_0=\pm\frac{\sqrt{\omega}}{\sqrt{-\kappa\left(2\kappa^2-5\right)\left(\gamma_j+\lambda_j\right)}},\ g_1=0,\ f_1=\mp\frac{2\sqrt{\omega}}{\sqrt{-\kappa\left(2\kappa^2-5\right)\left(\gamma_j+\lambda_j\right)}},$$

$$m=\frac{1}{2},\ a_j=\frac{2\omega}{2\kappa^2-5}$$

Then, the Jacobi elliptic function solutions of the Eqs. (7) and (8) are:

$$\psi(x,t)=\pm\frac{\sqrt{\omega}}{\sqrt{-\kappa\,\left(2\,\kappa^2-5\right)\,(\gamma_1+\lambda_1)}}\ \left(\frac{-1+3\operatorname{nc}[x-\rho t]^2}{1+\operatorname{nc}[x-\rho t]^2}\right) e^{i\,(-\kappa\,x\,+\,\omega\,t+\zeta\,)}, \quad (31)$$

$$\phi(x,t)=\pm\frac{\sqrt{\omega}}{\sqrt{-\kappa\,\left(2\,\kappa^2-5\right)\,(\gamma_2+\lambda_2)}}\ \left(\frac{-1+3\operatorname{nc}[x-\rho t]^2}{1+\operatorname{nc}[x-\rho t]^2}\right) e^{i\,(-\kappa\,x\,+\,\omega\,t+\zeta\,)}, \quad (32)$$

and $\kappa\,\omega\,\left(2\,\kappa^2-5\right)\left(\gamma_j+\lambda_j\right)<0\,,\ j=1,\ 2.$

Result 2

$$g_0=\pm\frac{\sqrt{\omega}}{\sqrt{\kappa\left(2\kappa^2+5\right)\left(\gamma_j+\lambda_j\right)}},\ g_1=0,\ f_1=\pm\frac{2\sqrt{\omega}}{\sqrt{\kappa\left(2\kappa^2+5\right)\left(\gamma_j+\lambda_j\right)}},$$

$$m=\frac{\sqrt{3}}{2},\ a_j=\frac{2\omega}{2\kappa^2+5}$$

Then, the Jacobi elliptic function solutions of the Eqs. (7) and (8) are:

$$\psi(x,t)=\pm\frac{\sqrt{\omega}}{\sqrt{\kappa\,\left(2\,\kappa^2+5\right)\,(\gamma_1+\lambda_1)}}\ \left(\frac{3-\operatorname{nc}[x-\rho t]^2}{1+\operatorname{nc}[x-\rho t]^2}\right)\ e^{i\,(-\kappa\,x\,+\,\omega\,t+\zeta\,)},$$

$$(33)$$

$$\phi(x,t)=\pm\frac{\sqrt{\omega}}{\sqrt{\kappa\ \left(2\,\kappa^2+5\right)\ (\gamma_2+\lambda_2)}}\ \left(\frac{3-\text{nc}[x-\rho t]^2}{1+\text{nc}[x-\rho t]^2}\right)\ e^{i\ (-\kappa\ x\ +\ \omega\ t+\zeta\ )}, \tag{34}$$

and $\kappa\ \omega\ \left(\gamma_j+\lambda_j\right)>0\ ,\ j=1,\ 2.$

Type 5 When $s=r=\frac{1}{4}\ ,\ c=\frac{1-2m^2}{2}$ and $z(\vartheta)=\text{ns}(\vartheta)\pm\text{cs}(\vartheta)$, we have two results: ***Result 1***

$$g_0=0,\quad g_1=\pm\frac{2\sqrt{2\ \left(\omega-\left(\kappa^2+1\right)a_j\right)}}{\sqrt{\kappa\ \left(\gamma_j+\lambda_j\right)}},\quad f_1=0,\quad m=\frac{\sqrt{\omega-a_j\left(\kappa^2+1\right)}}{\sqrt{a_j}}$$

Then, the Jacobi elliptic function solutions of the Eqs. (7) and (8) are:

$$\psi(x,t)=\pm\frac{2\sqrt{2\left(\omega-a_1\left(\kappa^2+1\right)\right)}}{\sqrt{\kappa\ (\gamma_1+\lambda_1)}}\ \left(\frac{\text{ns}[x-\rho\ t]\pm\text{cs}[x-\rho\ t]}{1+(\text{ns}[x-\rho\ t]\pm\text{cs}[x-\rho\ t])^2}\right)\ e^{i\ (-\kappa\ x+\omega\ t+\zeta)}, \tag{35}$$

$$\phi(x,t)=\pm\frac{2\sqrt{2\left(\omega-a_2\left(\kappa^2+1\right)\right)}}{\sqrt{\kappa\ (\gamma_2+\lambda_2)}}\ \left(\frac{\text{ns}[x-\rho\ t]\pm\text{cs}[x-\rho\ t]}{1+(\text{ns}[x-\rho\ t]\pm\text{cs}[x-\rho\ t])^2}\right)\ e^{i\ (-\kappa\ x+\omega\ t+\zeta)}, \tag{36}$$

with $\kappa\ \left(\gamma_j+\lambda_j\right)\left(\omega-a_j\ \left(\kappa^2+1\right)\right)>0$, $a_j\ \left(\kappa^2+1\right)<\omega\le a_j\ \left(\kappa^2+2\right)$ and $a_j>0\quad j=1,\ 2.$

Specifically, let $\omega=a_j\ \left(\kappa^2+2\right)$, then, the dark soliton solutions of the Eqs. (7) and (8) are:

$$\psi(x,t)=\pm\frac{\sqrt{2\,a_1}}{\sqrt{\kappa\ (\gamma_1+\lambda_1)}}\ \tanh[x-\rho\ t]\,e^{i\ (-\kappa\ x+\ \left(\kappa^2+2\right)\ a_1\ t+\zeta)}, \tag{37}$$

$$\phi(x,t)=\pm\frac{\sqrt{2\,a_2}}{\sqrt{\kappa\ (\gamma_2+\lambda_2)}}\ \tanh[x-\rho\ t]\,e^{i\ (-\kappa\ x+\ \left(\kappa^2+2\right)\ a_2\ t+\zeta)}. \tag{38}$$

Result 2

$$g_0=0,\quad g_1=0,\quad f_1=\pm\frac{\sqrt{\omega-\left(\kappa^2+1\right)a_j}}{\sqrt{\kappa\ \left(\gamma_j+\lambda_j\right)}},\quad m=\frac{\sqrt{-\omega+a_j\left(\kappa^2+1\right)}}{\sqrt{2\,a_j}}$$

Then, the Jacobi elliptic function solutions of the Eqs. (7) and (8) are:

$$\psi(x,t)=\pm\frac{\sqrt{\omega-a_1\left(\kappa^2+1\right)}}{\sqrt{\kappa\ (\gamma_1+\lambda_1)}}\ \left(\frac{1-(\text{ns}[x-\rho\ t]\pm\text{cs}[x-\rho\ t])^2}{1+(\text{ns}[x-\rho\ t]\pm\text{cs}[x-\rho\ t])^2}\right)$$

$$e^{i\ (-\kappa\ x+\omega\ t+\zeta)}, \tag{39}$$

$$\phi(x,t) = \pm\frac{\sqrt{\omega - a_2\left(\kappa^2+1\right)}}{\sqrt{\kappa\ \left(\gamma_2+\lambda_2\right)}}\ \left(\frac{1-(\text{ns}[x-\rho\ t]\pm\text{cs}[x-\rho\ t])^2}{1+(\text{ns}[x-\rho\ t]\pm\text{cs}[x-\rho\ t])^2}\right)$$
$$e^{i\ (-\kappa\ x+\omega\ t+\zeta)}, \tag{40}$$

with $\kappa\ \left(\gamma_j+\lambda_j\right)\ \left(\omega-a_j\ \left(\kappa^2+1\right)\right) > 0,\quad a_j\ \left(\kappa^2-1\right) \le \omega < a_j\ \left(\kappa^2+1\right)$ and $a_j > 0\quad j=1,\ 2.$
Specifically, let $\omega = a_j\ \left(\kappa^2-1\right)$, then, the bright soliton solutions of the Eqs. (7) and (8) are:

$$\psi(x,t) = \pm\frac{\sqrt{-2\ a_1}}{\sqrt{\kappa\ \left(\gamma_1+\lambda_1\right)}}\ \text{sech}[x-\rho\ t]\ e^{i\ \left(-\kappa\ x+\ \left(\kappa^2-1\right)\ a_1\ t+\zeta\right)}, \tag{41}$$

$$\phi(x,t) = \pm\frac{\sqrt{-2\ a_2}}{\sqrt{\kappa\ \left(\gamma_2+\lambda_2\right)}}\ \text{sech}[x-\rho\ t]\ e^{i\ \left(-\kappa\ x+\ \left(\kappa^2-1\right)\ a_2\ t+\zeta\right)}. \tag{42}$$

Type 6 When $s = r = \frac{1-m^2}{4}$, $c = \frac{1+m^2}{2}$ and $z(\vartheta) = \text{nc}(\vartheta)\pm\text{sc}(\vartheta)$ or $z(\vartheta) = \frac{\text{cn}(\vartheta)}{1\pm\text{sn}(\vartheta)}$, we have two results:
Result 1

$$g_0 = 0,\quad g_1 = \pm\frac{2\sqrt{\omega-\left(\kappa^2+1\right)a_j}}{\sqrt{\kappa\ \left(\gamma_j+\lambda_j\right)}},\quad f_1 = 0,\quad m = \frac{\sqrt{-\omega+a_j\left(\kappa^2+1\right)}}{\sqrt{2\ a_j}}$$

Then, the Jacobi elliptic function solutions of the Eqs. (7) and (8) are:

$$\psi(x,t) = \pm\frac{2\sqrt{\omega - a_1\left(\kappa^2+1\right)}}{\sqrt{\kappa\ \left(\gamma_1+\lambda_1\right)}}\ \left(\frac{\text{nc}[x-\rho\ t]\pm sc[x-\rho\ t]}{1+(\text{nc}[x-\rho\ t]\pm sc[x-\rho\ t])^2}\right)$$
$$e^{i\ (-\kappa\ x+\omega\ t+\zeta)}, \tag{43}$$

$$\phi(x,t) = \pm\frac{2\sqrt{\omega - a_2\left(\kappa^2+1\right)}}{\sqrt{\kappa\ \left(\gamma_2+\lambda_2\right)}}\ \left(\frac{\text{nc}[x-\rho\ t]\pm sc[x-\rho\ t]}{1+(\text{nc}[x-\rho\ t]\pm sc[x-\rho\ t])^2}\right)$$
$$e^{i\ (-\kappa\ x+\omega\ t+\zeta)}, \tag{44}$$

or

$$\psi(x,t) = \frac{2\sqrt{\omega - a_1\left(\kappa^2+1\right)}}{\sqrt{\kappa\ \left(\gamma_1+\lambda_1\right)}}\ \left(\frac{\text{cn}[x-\rho\ t]\,(\pm1+\text{sn}[x-\rho\ t])}{\text{cn}[x-\rho\ t]^2+(\pm1+sc[x-\rho\ t])^2}\right)$$
$$e^{i\ (-\kappa\ x+\omega\ t+\zeta)}, \tag{45}$$

$$\phi(x,t) = \frac{2\sqrt{\omega - a_2\left(\kappa^2+1\right)}}{\sqrt{\kappa\ \left(\gamma_2+\lambda_2\right)}}\ \left(\frac{\text{cn}[x-\rho\ t]\,(\pm1+\text{sn}[x-\rho\ t])}{\text{cn}[x-\rho\ t]^2+(\pm1+sc[x-\rho\ t])^2}\right)$$

$$e^{i\ (-\kappa\ x+\omega\ t+\zeta)}, \tag{46}$$

with $\kappa\ \left(\gamma_j+\lambda_j\right)\left(\omega-a_j\ \left(\kappa^2+1\right)\right)\ >\ 0,\ \ a_j\ \left(\kappa^2-1\right)\ \le\ \omega\ <\ a_j\ \left(\kappa^2+1\right)$ and $a_j>0\quad j=1,\ 2.$
Specifically, let $\omega=a_j\ \left(\kappa^2-1\right)$, then, the hyperbolic function solutions and bright soliton solutions of the Eqs. (7) and (8) are:

$$\psi(x,t)=\pm\frac{2\sqrt{-2a_1}}{\sqrt{\kappa\ (\gamma_1+\lambda_1)}}\ \left(\frac{\cosh[x-\rho t]+\tanh[x-\rho t]}{1+(\cosh[x-\rho t]+\tanh[x-\rho t])^2}\right)$$
$$e^{i\left(-\kappa x+\left(\kappa^2-1\right)a_1 t+\zeta\right)}, \tag{47}$$
$$\phi(x,t)=\pm\frac{2\sqrt{-2a_2}}{\sqrt{\kappa\ (\gamma_2+\lambda_2)}}\ \left(\frac{\cosh[x-\rho t]+\tanh[x-\rho t]}{1+(\cosh[x-\rho t]+\tanh[x-\rho t])^2}\right)$$
$$e^{i\left(-\kappa x+\left(\kappa^2-1\right)a_2 t+\zeta\right)}, \tag{48}$$

or

$$\psi(x,t)=\pm\frac{\sqrt{-2a_1}}{\sqrt{\kappa\ (\gamma_1+\lambda_1)}}\ \operatorname{sech}[x-\rho t]\ e^{i\left(-\kappa x+\left(\kappa^2-1\right)a_1 t+\zeta\right)}, \tag{49}$$
$$\phi(x,t)=\pm\frac{\sqrt{-2a_2}}{\sqrt{\kappa\ (\gamma_2+\lambda_2)}}\ \operatorname{sech}[x-\rho t]\ e^{i\left(-\kappa x+\left(\kappa^2-1\right)a_2 t+\zeta\right)}. \tag{50}$$

Result 2

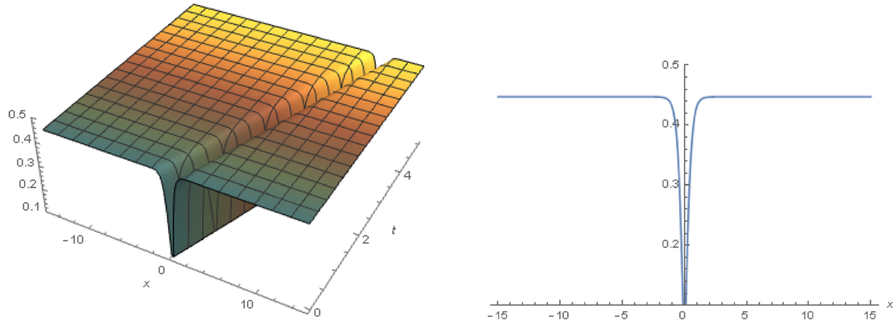
$$g_0=0,\quad g_1=0,\quad f_1=\pm\frac{\sqrt{2(\omega-(\kappa^2+1)a_j)}}{\sqrt{\kappa\ \left(\gamma_j+\lambda_j\right)}},\quad m=\frac{\sqrt{\omega-a_j\left(\kappa^2+1\right)}}{\sqrt{a_j}}$$

Then, the Jacobi elliptic function solutions of the Eqs. (7) and (8) are:

$$\psi(x,t)=\pm\frac{\sqrt{2\ \left(\omega-a_1\left(\kappa^2+1\right)\right)}}{\sqrt{\kappa\ (\gamma_1+\lambda_1)}}\ \left(\frac{1-(\mathrm{nc}[x-\rho\ t]\pm sc[x-\rho\ t])^2}{1+(\mathrm{nc}[x-\rho\ t]\pm sc[x-\rho\ t])^2}\right)$$
$$e^{i\ (-\kappa\ x+\omega\ t+\zeta)}, \tag{51}$$
$$\phi(x,t)=\pm\frac{\sqrt{2\ \left(\omega-a_2\left(\kappa^2+1\right)\right)}}{\sqrt{\kappa\ \ (\gamma_2+\lambda_2)}}\ \left(\frac{1-(\mathrm{nc}[x-\rho\ t]\pm sc[x-\rho\ t])^2}{1+(\mathrm{nc}[x-\rho\ t]\pm sc[x-\rho\ t])^2}\right)$$
$$e^{i\ (-\kappa\ x+\omega\ t+\zeta)}, \tag{52}$$

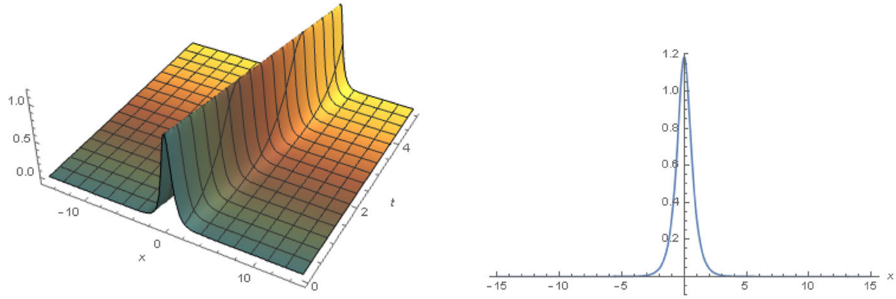
or

$$\psi(x,t)=\pm\frac{\sqrt{2\left(\omega-a_1\left(\kappa^2+1\right)\right)}}{\sqrt{\kappa\ (\gamma_1+\lambda_1)}}\ \left(\frac{-\,\mathrm{cn}[x-\rho t]+(\pm1+\mathrm{sn}[x-\rho t])^2}{\mathrm{cn}[x-\rho t]^2+(\pm1+sc[x-\rho t])^2}\right)$$
$$e^{i(-\kappa x+\omega t+\zeta)}, \tag{53}$$



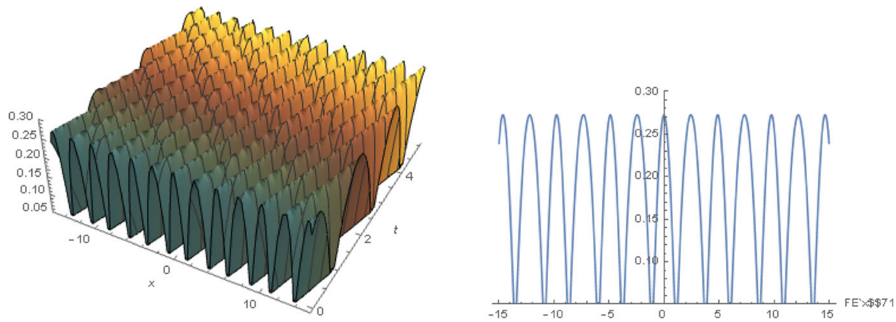
(a) (b)

Fig. 1 3D and 2D diagrams of a dark wave solution (19)



(a) (b)

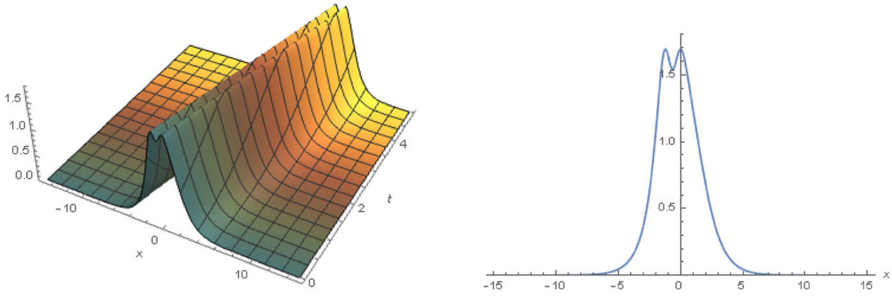
Fig. 2 3D and 2D diagrams of a bright wave solution (21)



(a) (b)

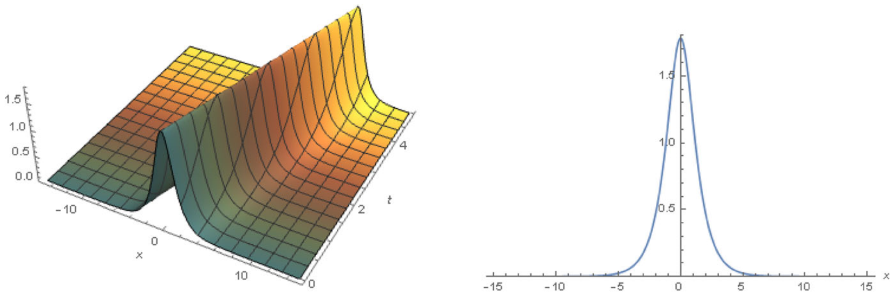
Fig. 3 3D and 2D diagrams of a Jacobi elliptic function solution (25)

$$\phi(x,t) = \pm\frac{\sqrt{2\left(\omega - a_2\left(\kappa^2+1\right)\right)}}{\sqrt{\kappa\left(\gamma_2+\lambda_2\right)}}\left(\frac{-\operatorname{cn}[x-\rho t]+(\pm 1+\operatorname{sn}[x-\rho t])^2}{\operatorname{cn}[x-\rho t]^2+(\pm 1+sc[x-\rho t])^2}\right)$$
$$e^{i(-\kappa x+\omega t+\zeta)}, \tag{54}$$



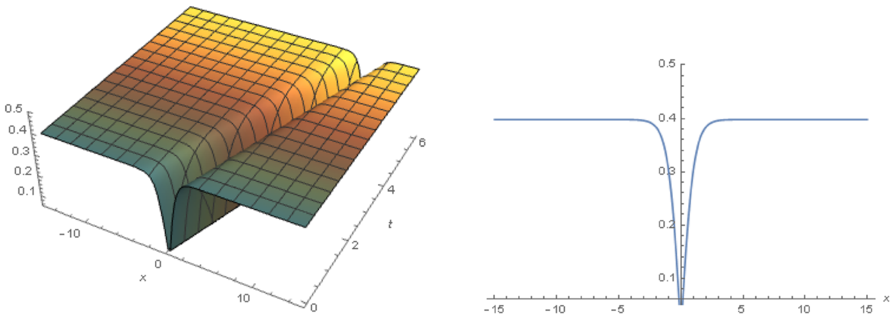
(a) (b)

Fig. 4 3D and 2D diagrams of a hyperbolic function solution (47)



(a) (b)

Fig. 5 3D and 2D diagrams of a bright wave solution (49)



(a) (b)

Fig. 6 3D and 2D diagrams of a dark wave solution (55)

with $\kappa\ (\gamma_j + \lambda_j)\left(\omega - a_j\ (\kappa^2 + 1)\right) > 0$, $a_j\ (\kappa^2 + 1) < \omega \le a_j\ (\kappa^2 + 2)$ and $a_j > 0 \quad j = 1,\ 2$.
Specifically, let $\omega = a_j\ (\kappa^2 + 2)$, then, the dark soliton solutions of the Eqs. (7) and (8) are:

$$\psi(x,t) = \mp \frac{\sqrt{2\, a_1}}{\sqrt{\kappa\ (\gamma_1 + \lambda_1)}}\ \tanh[x - \rho\ t]\, e^{i\ (-\kappa\ x + \ (\kappa^2+2)\ a_1\ t + \zeta)}, \tag{55}$$

$$\phi(x,t) = \mp\frac{\sqrt{2\ a_2}}{\sqrt{\kappa\ (\gamma_2+\lambda_2)}}\ \tanh[x-\rho\ t]\ e^{i\ (-\kappa\ x+\ (\kappa^2+2)\ a_2\ t+\zeta)}. \tag{56}$$

5 Graphic representation of solutions

In this part, some diagrams are presented in 3D and 2D format of some of the solutions obtained in this paper including solutions of fine travel waves and also of hyperbolic functions and other solutions in various shapes to fully understand this system. Figure 1 shows the 3D and 2D a dark wave solutions of Eq. (19) with $\omega = 1.2,\ \kappa = 2,\ \Upsilon_1 = \Upsilon_2 = 1.4,\ \lambda_1 = \lambda_2 = 0.6,\ \rho = 2,\ \zeta = 1$ and $-15 < x < 15$. Figure 2 shows the 3D and 2D a bright wave solution of Eq. (21) with $\omega = 1.4,\ \kappa = 1.2,\ \Upsilon_1 = \Upsilon_2 = 1,\ \lambda_1 = \lambda_2 = 1.6,\ \rho = 1,\ \zeta = 2$ and $-15 < x < 15$. Figure 3 shows the 3D and 2D a Jacobi elliptic function solution of Eq. (25) with $\omega = 1.3,\ \kappa = 1,\ \Upsilon_1 = \Upsilon_2 = 1.2,\ \lambda_1 = \lambda_2 = 1.3,\ \rho = 1.5,\ \zeta = 2$ and $-15 < x < 15$. Figure 4 shows the 3D and 2D a hyperbolic function solution of Eq. (47) with $\omega = 1.5,\ \kappa = 0.5,\ \Upsilon_1 = \Upsilon_2 = 1.3,\ \lambda_1 = \lambda_2 = 1.5,\ \rho = 1.2,\ \zeta = 1.3$ and $-15 < x < 15$. Figure 5 shows the 3D and 2D a bright soliton solution of Eq. (49) with $\omega = 1.4,\ \kappa = 0.6,\ \Upsilon_1 = \Upsilon_2 = 1.1,\ \lambda_1 = \lambda_2 = 1.2,\ \rho = 1.5,\ \zeta = 1.4$ and $-15 < x < 15$. Figure 6 shows the 3D and 2D a dark soliton solution of Eq. (55) with $\omega = 1.5,\ \kappa = 1.6,\ \Upsilon_1 = \Upsilon_2 = 1.4,\ \lambda_1 = \lambda_2 = 1.2,\ \rho = 1.5,\ \zeta = 1.4$ and $-15 < x < 15$.

6 Conclusion

The KN equation without four-wave mixing (FWM) terms in birefringent fibers has been studied successfully using Jacobi elliptic function expansion method. We obtained all possible solitons for this equation. Beside bright and dark solitons, Jacobi elliptic function solutions and hyperbolic solutions are also reported. Furthermore, 3D and 2D graphs of bright and dark soliton and Jacobi elliptic function solutions are presented for better illustration.

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Conflict of interest The authors declare that they have no competing interests.

References

1. Anderson, D., Lisak, M.: Nonlinear asymmetric self-phase modulation and self-steepening of pulses in long optical waveguides. Phys. Rev. A **27**(3), 1393–1398 (1983)
2. Biswas, A.: Soliton perturbation theory for Alfven waves in plasmas. Phys. Plasmas **12**(2), 022306 (2005)
3. Guha, P.: Geometry of the Kaup–Newell equation. Rep. Math. Phys. **50**(1), 1–12 (2002)
4. Jin-Chun, H., Zong-Yun, C.: Comment on revision of Kaup–Newell's works on IST for DNLS equation. Commun. Theor. Phys. **50**(6), 1369–1374 (2008)

5. Guo, H., Zhang, X., Ma, G., Zhang, X., Yang, C., Zhou, Q., Liu, W.: Analytic study on interactions of some types of solitary waves. Optik **164**, 132–137 (2018)
6. Ma, W.X.: A coupled AKNS-Kaup-Newell soliton hierarchy. J. Math. Phys. **40**, 4419 (1999)
7. Pashaev, O.K., Lee, J.-H.: Relativistic DNLS and Kaup–Newell hierarchy. Symm. Integr. Geom. Methods Appl. **13**, 58 (2017)
8. Tzoar, N., Jain, M.: Self-phase modulation in long-geometry optical waveguides. Phys. Rev. A **23**(3), 1266–1270 (1981)
9. Yang, J.-Y., Ma, W.X.: Conservation laws of a perturbed Kaup–Newell equation. Mod. Phys. Lett. B **30**, 1650381 (2016)
10. Zhihua, Y., Yunbo, Z.: On generating equations for the Kaup–Newell hierarchy. Appl. Math. A J. Chin. Univ. **22**(4), 413–420 (2007)
11. Hong-cai, M.A., Zhang, Z.-P., Deng, A.-P.: A new periodic solution to Jacobi elliptic functions of MKdV equation and BBM equation. Acta Math. Appl. Sin. **28**, 409 (2013)
12. El-Sheikh, M.M.A., Ahmed, H.M., Arnous, A.H., Rabie, W.B.: Optical solitons and other solutions in birefringent fibers with Biswas–Arshed equation by Jacobi's elliptic function approach. Optik **202**, 163546 (2020)
13. El-Sheikh, M.M.A., Seadawy, A.R., Ahmed, H.M., Arnous, A.H., Rabie, W.B.: Dispersive and propagation of shallow water waves as a higher order nonlinear Boussinesq-like dynamical wave equations. Phys. A **537**, 122662 (2020)
14. Arshed, S., Biswas, A., Abdelaty, M., Zhou, Q., Moshokoa, S.P., Belic, M.: Sub pico-second chirp-free optical solitons with Kaup–Newell equation using a couple of strategic algorithms. Optik **172**, 766–71 (2018)
15. Biswas, A., Yildirim, Y., Yasar, E., Zhou, Q., Moshokoa, S.P., Belic, M.: Sub-pico second pulses in mono-mode optical fibers with Kaup–Newell equation by a couple of integration schemes. Optik **167**, 121–8 (2018)
16. Biswas, A., Ekici, M., Sonmezoglu, A., Alqahtani, R.T.: Sub - pico second chirped optical solitons in mono - mode fibers with Kaup–Newell equation by extended trial function method. Optik **168**, 208–16 (2018)
17. Jawad, A.J.A.M., Al Azzawi, F.J.I., Biswas, A., Khan, S., Zhou, Q., Moshokoa, S.P., Belic, M.R.: Bright and singular optical solitons for Kaup-Newell equation with two fundamental integration norms. Optik **182**, 594–7 (2019)
18. Kudryashov, N.A.: Traveling wave solutions of the generalized nonlinear Schrodinger equation with cubic-quintic nonlinearity. Optik **188**, 27–35 (2019)
19. Kudryashov, N.A.: A generalized model for description of propagation pulses in optical fiber. Optik **189**, 42–52 (2019)
20. Kudryashov, N.A.: Construction of nonlinear differential equations for description of propagation pulses in optical fiber. Optik **192**, 162964 (2019)
21. Kudryashov, N.A.: General solution of traveling wave reduction for the Kundu–Mukherjee–Naskar model. Optik **186**, 22–7 (2019)
22. Kudryashov, N.A.: General solution of the traveling wave reduction for the perturbed Chen–Lee–Liu equation. Optik **186**, 339–49 (2019)
23. Kudryashov, N.A.: Solitary and periodic waves of the hierarchy for propagation pulse in optical fiber. Optik **194**, 163060 (2019)
24. Kudryashov, N.A.: First integrals and solutions of the traveling wave reduction for the Triki-Biswas equation. Optik **185**, 275–81 (2019)

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