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Title: A convex integration approach to classical non-uniqueness of characteristic curves of Sobolev vector fields

Abstract: The celebrated DiPerna-Lions theory guarantees the existence and uniqueness of regular Lagrangian flows for Sobolev vector fields. The latter are defined as a selection of trajectories of the related ODE satisfying additional compressibility properties. A long-standing open question is whether the uniqueness of the regular Lagrangian flow is a corollary of the uniqueness of the trajectory of the ODE for a.e. initial datum.

In this talk, I will explain how a new ill-posedness result for positive solutions to the continuity equation provides a negative answer to this question. Our technique builds upon a convex integration scheme introduced in this setting by Modena and Székelyhidi.

This is based on a joint work with M. Colombo and C. De Lellis.